# Interferometric Noise in Optical Time Division Multiplexing Transmission System

Jianfeng Zhang, Minyu Yao, Qianfan Xu, Hongming Zhang, Can Peng, and Yizhi Gao

*Abstract*—A theoretical assessment of the interferometric noise in the optical time division multiplexing (OTDM) transmission system is presented. The impact of such noise on the system performance were investigated in detail. The corresponding system requirements were also revealed by the calculation results. Recommendations on the use of prechirping to reduce the interferometric noise in the OTDM system are given.

*Index Terms*—Bit error rate, interferometric crosstalk, optical noise, optical time division multiplexing (OTDM), phase noise.

# I. INTRODUCTION

**O** PTICAL time division multiplexing (OTDM) is an efficient technique of increasing the capacity of optical transmission systems and optical networks. The performance of OTDM system is limited by a complex combination of noise including the interferometric noise [1], [2]. While the interferometric noise in the WDM networks has long attracted attention [9]–[11], only several papers report the investigations on such noise in the OTDM system in the recent years. Jepsen *et al.* [3] showed both theoretically and experimentally that the interferometric noise would become a serious problem to degrade the system performance when the pulses in different channels overlapped. Several methods were also demonstrated to suppress such noise in the OTDM data link by using non-linear switching [4], or filtering [5], special polarization control [6], or the transmultiplexing method [7].

However, a complete theoretical characterization of such noise in OTDM system and evaluation of its impact on the system performance were scarcely addressed until now. The aim of this paper is to characterize the interferometric noise in the OTDM transmission system and analyze the system requirements set by such noise, which is very necessary for the OTDM system design and optimization.

In Section II, the properties of the interferometric noise in OTDM transmission system are analyzed and the corresponding system model is presented. In Section III, the system performance under such noise is investigated in detail. The system requirements on the optical transmitter and the limitations on the transmission link imposed by the noise are analyzed. Discussions on the employment of prechirping technique to ease the

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system requirements are also presented. In Section IV, summarizing conclusions are given.

## II. INTERFEROMETRIC NOISE

The mechanism of how the interferometric noise is induced in an OTDM link is illustrated in Fig. 1. In a practical OTDM transmitter configuration, the optical pulses emitted by the laser source are interleaved into a number of channels and then modulated by the data. At the receiver, one channel will be selected by the optical demultiplexer and converted into electrical signal by the receiver. If the pulses have an unperfected extinction ratio or unsuitable pulsewidth, the pulses in different channels will overlap each other at the receiver. Under this condition, the multiplexer acts as an interferometer, so the received optical intensity will depend on the relative phase difference between the interfering channels. Usually such a phase difference fluctuates due to the phase noise existing in the pulse source, thus, the demultiplexed channel will be added by the intensity noise. In most cases, the interferometric noise falls inside the receiver bandwidth, which degrade the system performance greatly.

To characterize the interferometric noise in detail, the variance of the received signal power including two interfering channels is calculated in the following. The optical field in one channel is given by

$$E(t) = A(t)e^{i\varphi(t)}\vec{r}$$
<sup>(1)</sup>

where A(t) represents the field's envelope,  $\varphi(t)$  represents the random process of phase noise,  $\vec{r}$  represents the polarization state. With the assumption that the interfering signals have the matched polarization, the normalized received power at the time t can be given by

$$I = \frac{1}{T} \int_{t-T}^{t} [a_1 A^2(\tau) + a_2 A^2(\tau - \tau_d) + 2\sqrt{a_1 a_2} A(\tau) \\ \times A(\tau - \tau_d) \cos(\omega_0 \tau_d + \varphi(\tau) - \varphi(\tau - \tau_d))] \cdot W(\tau) \, d\tau \quad (2)$$

where  $\tau_d$  is the path delay time between two channels,  $W(\tau)$  is the switching window function of the demultiplexer, T is the bit interval at the base rate, and  $a_{1,2}$  represents the information bit in the corresponding channel.

In the derivation, the electrical receiver filtering process is approximated as integration over one bit interval. To make discussions clear, other possible system noise sources [2], such as the random timing jitter of the pulses, are not included in the expression.

For the chirp-free pulse source,  $\Delta \varphi(t) = \varphi(t) - \varphi(t - \tau_d)$  can be approximated as a zero-mean Gaussian process of Wiener



Fig. 1. Interferometric noise in OTDM transmission system. (a) OTDM transmitter. (b) OTDM receiver.

form [11], [13], the expectation and variance of which is given by

$$E(\cos(\beta + \Delta\varphi(t)) = \cos(\beta)e^{-\frac{\tau_d}{2\tau_c}}$$
(3)

$$E(\cos^2(\beta + \Delta\varphi(t)) = 0.5(1 + \cos(2\beta)e^{-\frac{2\tau_d}{\tau_c}}).$$
 (4)

 $\beta$  is an arbitrary phase value,  $\tau_c$  denotes the coherence time of the laser source, which provides the characterization of phase fluctuations. For most pulse sources employed in OTDM system,  $\tau_c$  ranges from several nanoseconds to a few 100 ns, while T is 100 ps or less in high-bit-rate system. Thus,  $\Delta\varphi(\tau)$ function in (2) is regarded as a slowly varying function, and the third term in this expression can be further simplified

$$\Delta I = \frac{1}{T} \int_{t-T}^{t} [2\sqrt{a_1 a_2} A(\tau) A(\tau - \tau_d) \\ \times \cos(\omega_0 \tau_d + \Delta \phi(\tau))] W(\tau) d\tau$$
$$\approx \frac{1}{T} \int_{t-T}^{t} [2\sqrt{a_1 a_2} A(\tau) A(\tau - \tau_d)] \\ \times W(\tau) d\tau \cdot \cos(\omega_0 \tau_d + \Delta \phi(t)). \tag{5}$$

Using (3)–(5), the expectation and variance of the signal power fluctuation can be calculated for  $(a_1, a_2) \in (1, 1)$ 

$$E[\Delta I] = I_0 \cos(\omega_0 \tau_d) \cdot e^{\frac{-\tau_d}{2\tau_c}}$$
(6a)  

$$\sigma_I^2 = E(I^2) - E^2(I)$$
  

$$= I_0^2 \left[ 0.5 \left( 1 + \cos(2\omega_0 \tau_d) \cdot e^{-\frac{2\tau_d}{\tau_c}} \right)$$
  

$$- \cos^2(\omega_0 \tau_d) e^{-\frac{\tau_d}{\tau_c}} \right]$$
(6b)

where  $I_0 = (2/T) \int_t^{t-T} A(\tau) A(\tau - \tau_d) W(\tau) d\tau$  denotes the crosstalk power. Equations (6a) and (6b) show that  $\tau_d/\tau_c$  is an important parameter. For the large value of  $\tau_d/\tau_c$ , the average signal power is irrelevant to the relative phase difference  $\omega \tau_d$  between the two interfering fields. In other words, the interfering



Fig. 2. Variance of interferometric noise as a function of  $\tau_d/\tau_c$ .

fields combine incoherently. In this incoherent regime, the maximum conversion of phase noise into intensity noise occurs and the variance is also irrelevant to  $\omega \tau_d$ , as shown in Fig. 2. While for the small value of  $\tau_d/\tau_c \ll 1$ , the expectation is strongly dependent on the value of  $\omega \tau_d$ , thus, the fields combine coherently. In this regime, variance approaches the maximum value when fields interfere in quadrature ( $\omega \tau_d = n\pi + \pi/2$ ) and the minimum value when they interfere in phase ( $\omega \tau_d = n\pi + \pi$ ). Slow fluctuations in  $\tau_d$  induced by environmental perturbations will result in fluctuations of both signal and noise power, which may bring about the system instability. Note, that when the value of  $\tau_d/\tau_c$  is approximately zero, the interferometric noise would disappear. Such can be realized by carefully configuring OTDM multiplexers, which may need hybrid integration with intensity modulators regarding the practical use of OTDM system [18].

As for a usual OTDM multiplexer, the path delays between channels vary from several centimeters to several meters, which determines the system operating in the semicoherent or coherent regime. While for an OTDM add/drop multiplexer (ADM), the pulses in the added channel is emitted by a different pulse source and have uncorrelated phase to other channels, the system operates in the incoherent regime.

In the following, we evaluated the system requirements set by the interferometric noise in terms of the incoherent regime, where the noise power approaches the maximum value (the worst case):

$$\Delta\varphi(t) = \varphi(t) - \varphi(t - \tau_d) = \Phi(t) + \Delta\varphi \tag{7}$$

where  $\Phi(t)$  is the regular phase relationship function between channels and  $\Delta \varphi$  represents the random variable of the phase noise, which varies uniformly between  $[-\pi, \pi]$ . Then, several parameters are defined in the following:

$$m = \frac{1}{T} \int_{0}^{T} A^{2}(\tau) W(\tau) d\tau$$
(8a)

$$\delta = \frac{\int_0^T A^2(\tau - \tau_d) W(\tau) \, d\tau}{\int_0^T A^2(\tau) W(\tau) \, d\tau}$$
(8b)

$$\sqrt{\varepsilon} = \frac{\int_0^T A(\tau - \tau_d) A(\tau) \cos(\Phi(\tau)) W(\tau) \, d\tau}{\int_0^T A^2(\tau) W(\tau) \, d\tau} \qquad (8c)$$

where *m* represents the average received signal power,  $\delta$  represents the intersymbol crosstalk from the neighboring channel, and  $\varepsilon$  represents the interferometric crosstalk, which indicates the effect of interferometric noise on the system. Substituting (2) and (7) into (8), the received power at the decision time (t = 0) can be simplified as the following:

$$I = m(a_1 + \delta \cdot a_2 + 2\sqrt{a_1 a_2} \cdot \sqrt{\varepsilon}). \tag{9}$$

Due to the fact that the number of channels contributing to the interferometric noise is small, the noise distribution deviates far from the Gaussian distribution [8]. We adopt the system model based on the moment generation function presented in reference [2] to accurately evaluate such noise. Then its corresponding conditioned MGF can be expressed by

$$M_{I}(s) = E[e^{sI}] = e^{sm(a_{1}+a_{2}\cdot\delta)} \cdot E\left[e^{2s\sqrt{a_{1}a_{2}}\sqrt{\varepsilon}\cdot\cos(\Delta\phi)}\right]$$
$$= e^{sm(a_{1}+a_{2}\cdot\delta)} \cdot I_{0}(2\sqrt{a_{1}a_{2}}\cdot\sqrt{\varepsilon}\cdot s).$$
(10)

For more than two interfering terms, the conditioned MGF can be derived in the same way

$$M_{\Lambda}(s) = e^{sma_1} \prod_{i=2}^{N} e^{sma_i \cdot \delta_i} I_0(\sqrt{a_1(t)a_i(t)\varepsilon_i} \cdot s)$$
(11)

where  $\varepsilon_i$  and  $\delta_i$  represent the intersymbol crosstalk and interferometric crosstalk from the *i*th channel.  $I_0(\cdot)$  is the zero-order modified Bessel function of the first kind.

The converted signal is also corrupted by the electrical thermal noise originating in the electrical receiver, so the MGF for the total decision variable Z is given by

$$M_z(s) = E_a[M_I(s)] \cdot e^{-\sigma_{\rm th}^2 s^2/2}$$
 (12)

where  $\sigma_{\rm th}$  represents electrical noise and the operator  $E_a[\cdot] = (1/2^{N-1}) \sum_{a_2...a_N} [\cdot]$  represents the statistical averaging over the bit symbols.



Fig. 3. System performance in the presence of interferometric crosstalk and intersymbol crosstalk.

Several methods have been proposed to apply to MGF to evaluate the system BER, such as modified Chernoff bound (MCB) [14] and saddlepoint approximation (SPA) [12]. In this paper, we apply SPA on (12) to evaluate the noise performance, which can result in satisfactory accuracy. In the following calculations, the switching window is assumed to be square and equal to the width of the channel slot (ideal), and the power penalty is calculated at BER =  $10^{-9}$ .

### **III. PERFORMANCE ANALYSIS**

## A. Transmitter

Clearly, the pulses generated by the pulse source should not overlap to avoid interference between adjacent channels. Thus, the optical pulse source is a key element determining the whole system's performance. In this section, the impact of channel interference and the system requirements on the pulse source will be generally analyzed. In the following discussions, we assume that the pulses are chirpless and only neighboring channels contribute to the channel crosstalk. such a assumption is reasonable in most cases.

In Section II, it was shown that overlapping of channels will bring about both the interferometric crosstalk ( $\varepsilon$ ) and intersymbol crosstalk ( $\delta$ ). Fig. 3 shows the system performance in the presence of both crosstalks. We notice that the performance degradation is more sensitive to the interferometric crosstalk.  $\varepsilon$  should not be larger than -24.4 dB to keep the system performance in the tolerable range (<1 dB), while the system performance indicates little difference when the intersymbol crosstalk varies between -30 and -15 dB. Calculations show that the resulted intersymbol crosstalk  $\delta$  would not be larger than -15 dB for a usual OTDM transmitter, thus, interferometric noise play the major role in degrading the system performance. In fact, with the following approximation:

$$\varepsilon = \left(\frac{\int_0^T A(t - \tau_d) A(t) W(t) \, dt}{\int_0^T A^2(t) W(t) \, dt}\right)^2 \approx \frac{\int_0^T A^2(t - \tau_d) A^2(t) \, dt}{\int_0^T A^4(t) \, dt}$$
(13)



Fig. 4. Parameters used for pulse characterization.



Fig. 5. Power penalty versus the pulses with different pulsewidth and extinction ratio.

 $\varepsilon$  is a measurable parameter, which can be estimated by an intensity autocorrelator.

Furthermore, two parameters are used to characterize the pulses emitted by the laser source, the FWHM pulsewidth  $\tau_{\rm FWHM}$  and pulse extinction ratio ER, which is illustrated in Fig. 4. The impacts of these two parameters on the system performance are calculated, which is shown in Fig. 5. The presented results are corresponding to pulses with secant shape and Gaussian shape.

As we see, the pulse pedestal has a very damaging effect on the system performance. Higher than 28 dB ER should be achieved to keep the penalty below 1 dB. It is also shown that different requirements on the duty ratio are imposed to keep system in good performance for pulses with different ERs. High extinction ratio would ease the requirements on the duty ratio. In general, duty ratio should be kept below 0.3 for secant shape while 0.4 for Gaussian shape. The difference stems from the fact that secant shape has a more weight in its tail than the Gaussian one.

# B. Propagation in Dispersive Fiber

As for the high-speed OTDM link in the linear or pseudolinear transmission regime [6], accurate dispersion compensation becomes very necessary. The dispersion mismatch would broaden the pulses and contribute to crosstalk between the channels. However, such channel overlapping is different with that discussed in Section III-A. This is due to the fact that the pulses



Fig. 6. (a) Channel crosstalk. (b) System penalty against the accumulated dispersion.

have a linear chirp imposed by the chromatic dispersion and the system performs in a different way. Investigations on the interferometric noise arising from overlapped pulses resulted by the dispersion mismatch is very necessary for understanding the whole system performance. Especially, such a investigation also reveals the dispersion tolerance of the transmission link. In this section, we take the  $4 \times 10$  Gb/s OTDM transmission link for the example to show the impacts of the noise.

Utilizing (8b) and (8c), Fig. 6 shows the relation between the channel crosstalk and the net dispersion. In the calculation, the fiber nonlinear effect is neglected, the pulsewidth and pulse extinction ratio are assumed to be 8 ps and -35 dB, respectively. As illustrated in Fig. 6, the pulsewidth increases quickly with the dispersion and result in overlapping of the channels. The overlapping effectively increases the intersymbol crosstalk  $\delta$ , while only increase the interferometric crosstalk a little (no more 1 dB for 40 ps/nm in this figure). This is due to the fact that the chirp in the pulses introduces the frequency difference between the interfering channels, and moves much of interferometric noise outside the receiver filter.

Therefore, the performance degradation is mainly attributed to the increased intersymbol crosstalk in the presence of interferometric noise. Consequently, we can conclude that the intersymbol crosstalk dominates concerning the channel overlapping resulted by the dispersion mismatch.



Interferometric noise power spectra

Fig. 7. Principle of reduction of interferometric noise by prechirping.



Fig. 8. System penalty against the phase modulation depth.

The limitation of the both crosstalks on the dispersion tolerance is illustrated in Fig. 6(b). As for 5-ps pulses, the dispersion range within 1 dB power penalty is +30 ps/nm, while for 12-ps pulses, +55 ps/nm. Clearly, pulses with a wider pulsewidth have a narrower spectral width, thus, have greater dispersion tolerance. However, the initial pulsewidth can not be too large due to the limitation of the interferometric noise. This result reveals a rule for the selection of pulsewidth concerning the dispersion tolerance of OTDM link.

#### C. Interferometric Noise Reduction by Prechiping

Prechirping is an effective technique to suppress the distortion induced by fiber nonlinearity in long-haul transmission [16], [17]. In this section, we show that prechirping can be employed to reduce the interferometric noise and ease the system requirements on the pulse source in the OTDM system. The principle is illustrated in Fig. 7. The phase modulator imposes the chirp on the pulse and, thus, introduces a frequency difference between the leading and trailing edges. As for interfering channels, part of the interferometric noise fall outside the receiver filter bandwidth, and the interferometric noise can be reduced in this way.

Assuming the phase modulation function is given by  $\Phi = p\cos(2\pi ft)$ , we can see the effect of noise reduction is dependent on the phase modulation depth p in Fig. 8. For p = 2,

the requirements on the pulse extinction ratio has been relaxed by more than 12 dB. Under this condition, pulses with only 16-dB ER can be employed in the OTDM system. Therefore, prechirping technique make it possible for the commercial modulators (LiNbO<sub>3</sub> or EA) to be employed in the OTDM system as the optical pulse sources.

However, prechirping simultaneously broadens the pulse spectrum, which may decrease the spectrum efficiency of the OTDM/WDM system. Thus, the phase modulation depth should be selected reasonably for practical system applications.

# **IV. CONCLUSION**

The system model for the interferometric noise in OTDM transmission system is presented. The relative system requirements are investigated in detail. Prechirping is shown to be a effective technique to ease the system requirements on the pulse source.

#### REFERENCES

- K. Uchiyama, T. Morioka, S. Kawanishi, H. Takara, and M. Saruwatari, "Signal-to-noise ratio analysis of 100 gb/s demultiplexing using nonlinear optical loop mirror," *J. Lightwave Technol.*, vol. 15, pp. 194–201, Feb. 1997.
- [2] J. F. Zhang, M. Y. Yao, X. Z. Chen, L. X, M. H. Chen, and Y. Z. Gao, "Bit error rate analysis of OTDM system based on moment generation function," *J. Lightwave Technol.*, vol. 18, pp. 1513–1518, Nov. 2000.
- [3] K. S. Jepsen, H. N. Poulsen, A. T. Clausen, A. Buxens, and K. E. Stubkjaer, "Investigation of cascadability of add-drop multiplexers in OTDM systems," in *Proc. ECOC*, 1998, pp. 619–620.
- [4] I. Y. Khrushchev, I. D. Phillips, A. D. Ellis, R. J. Manning, D. Nesset, D. G. Moodie, R. V. Penty, and I. H. White, "OTDM applications of dispersion-imbalanced fiber loop mirror," *Electron. Lett.*, vol. 35, no. 14, pp. 1183–1185, July 1999.
- [5] M. L. Nielsen, B. Olsson, and D. J. Blumenthal, "Pulse extinction ratio improvement using SPM in an SOA for OTDM systems applications," presented at the *Proc. ECOC*, 2000.
- [6] B. Mikkelsen, G. Raybon, and R.-J. Essiambre, "160 Gb/s TDM transmission systems," presented at the *Proc. ECOC*, 2000.
- [7] H. Tanaka, M. Hayashi, T. Otani, K. Ohara, and M. Suzuki, "60 Gb/s WDM-OTDM transmultiplexing using an electroabsorption modulator," presented at the *Proc. OFC*, 2001. Paper ME4.
- [8] D. D. Marcenac, A. D. Ellis, and D. G. Moodie, "80 Gb/s OTDM using electroabsorption modulators," *Electron. Lett.*, vol. 34, no. 1, pp. 101–103, Jan. 1998.
- [9] I. T. Monroy, E. Tangdiongga, and H. de Waardt, "Interferometric crosstalk reduction by phase scrambling," *J. Lightwave Technol.*, vol. 18, pp. 637–646, May 2000.
- [10] I. Monroy, E. Tangdiongga, and E. de Waardt, "On the distribution and performance implications of filtered interferometric crosstalk in optical WDM networks," *J. Lightwave Technol.*, vol. 17, pp. 989–997, June 1999.
- [11] P. K. Pepeljugoski and K. Y. Lau, "Interferometric noise reduction in fiber-optic links by superposition of high frequency modulation," *J. Lightwave Technol.*, vol. 10, pp. 957–963, July 1992.
- [12] G. Einarsson, *Principles of Lightwave Communications*. New York: Wiley, 1996.
- [13] A. Mooradian, "Laser linewidth," *Phys. Today*, vol. 38, pp. 43–48, May 1985.
- [14] I. T. Monory and E. Tangdiongga, "Performance evaluation of optical cross-connects by saddlepoint approximation," J. Lightwave Technol., vol. 16, pp. 317–323, Mar. 1998.
- [15] J. J. O'Reilly and R. F. Rocha, "Improved error probability evaluation methods for direct detection optical communication systems," *IEEE Trans. Inform. Theory*, vol. IT-33, pp. 839–848, June 1987.
- [16] A. Sano, Y. Miyamoto, S. Kuwahara, and H. Toba, "40-Gb/s/ch WDM transmission with SPM/XPM suppression through prechirping and dispersion management," *J. Lightwave Technol.*, vol. 18, pp. 1519–1524, Nov. 2000.

- [17] E. A. Golovchenko, A. N. Pilipetskii, and N. S. Bergano, "Transmission properties of chirped return-to-zero pulses and nonlinear intersymbol interference in 10 Gb/s WDM transmission," in *Proc. OFC Tech. Dig.*, vol. 4, 2000, pp. 38–40.
- [18] F. Zamkotsian, K. Sato, H. Okamoto, K. Kishi, I. Kotaka, M. Yamamoto, Y. Kondo, H. Yasaka, Y. Yoshikuni, and K. Oe, "Generation and coding of a 100 Gb/s signal by an InP-based optical multiplexer integrated with modulators," *Electron. Lett.*, vol. 31, no. 7, pp. 578–579, Mar. 1995.

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